**Chapter 3**

**Techniques of Integration**

**3.7 Improper Integrals**

**Section Exercises**

**Evaluate the following integrals. If the integral is not convergent, answer “divergent.”**

347. 

Answer: divergent

349. 

Answer: 

351. 

Answer: 

353. Without integrating, determine whether the integral  converges or diverges by comparing the function  with 

Answer: Converges

**Determine whether the improper integrals converge or diverge. If possible, determine the value of the integrals that converge.**

355. 

Answer: Converges to 1/2.

357. 

Answer: –4

359. 

Answer: 

361. 

Answer: diverges

363. 

Answer: diverges

365. 

Answer: 1.5

367. 

Answer: diverges

369. 

Answer: diverges

371. 

Answer: diverges

**Determine the convergence of each of the following integrals by comparison with the given integral. If the integral converges, find the number to which it converges.**

373.  compare with 

Answer: Both integrals diverge.

**Evaluate the integrals. If the integral diverges, answer “diverges.”**

375. 

Answer: diverges

377. 

Answer: diverges

379. 

Answer: 

381. 

Answer: 0.0

383. 

Answer: 0.0

**Evaluate the improper integrals. Each of these integrals has an infinite discontinuity either at an endpoint or at an interior point of the interval.**

385. 

Answer: 6.0

387. 

Answer: 

389. 

Answer: 

391. Evaluate  (Be careful!) (Express your answer using three decimal places.)

Answer: 

393. Evaluate 

Answer: 

395. Find the area of the region bounded by the curve  the *x*-axis, and on the left by 

Answer: 7.0

397. Find the area under  in the first quadrant.

Answer: 

399. Find the volume of the solid generated by revolving about the *y*-axis the region under the curve  in the first quadrant.

Answer: 

**The Laplace transformof a continuous function over the interval  is defined by  (see the Student Project). This definition is used to solve some important initial-value problems in differential equations, as discussed later. The domain of *F* is the set of all real numbers *s* such that the improper integral converges. Find the Laplace transform *F* of each of the following functions and give the domain of *F*.**

401. 

Answer: 

403. ]

Answer: 

405. Use the formula for arc length to show that the circumference of the circle  is 

Answer: Answers will vary.

**A function is a probability density function if it satisfies the following definition:  The probability that a random variable *x* lies between *a* and *b* is given by **

407. Find the probability that *x* is between 0 and 0.3. (Use the function defined in the preceding problem.) Use four-place decimal accuracy.

Answer: 0.8775

**Chapter Review Exercises**

**For the following exercises, determine whether the statement is true or false. Justify your answer with a proof or a counterexample.**

409. cannot be integrated using partial fractions.

Answer: False

411. Integration by parts can always yield the integral.

Answer: False

**For the following exercises, evaluate the integral using the specified method.**

413.  using trigonometric substitution

Answer: 

415.  using partial fractions

Answer: 

417.  using a table of integrals or a CAS

Answer: 

**For the following exercises, integrate using whatever method you choose.**

419. 

Answer: 

421. 

Answer: 

**For the following exercises, approximate the integrals using the midpoint rule, trapezoidal rule, and Simpson’s rule using four subintervals, rounding to three decimals.**

423. **[T]** 

Answer: 

425.  **[T]** 

Answer: 

**For the following exercises, evaluate the integrals, if possible.**

427. 

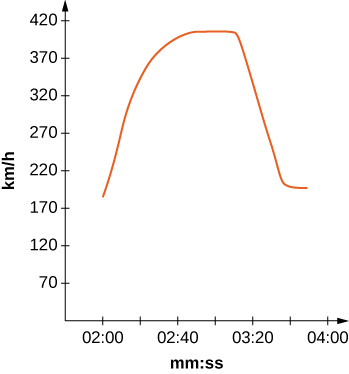
Answer: approximately 0.2194

**For the following exercises, consider the gamma function given by **

429. Extend to show that  assuming is a positive integer.

Answer: This is a proof; therefore, no answer is provided.

**The fastest car in the world, the Bugati Veyron, can reach a top speed of 408 km/h. The graph represents its velocity.**



431. **[T]** Using your function from the previous problem, find exactly how far the Bugati Veyron traveled in the 1 min 40 sec included in the graph.

Answer: Answers may vary. Ex:km

**Student Project**

**Laplace Transforms**

1. Calculate the Laplace transform of 

Answer: 

3. Calculate the Laplace transform of  (Note, you will have to integrate by parts twice.)

Answer: 

5. Use integration by parts to evaluate  (Let  and  Note, by the way, that we have defined  

Answer: This is a proof; therefore, no answer is provided.

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